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## Homogenization of planar elastic checkerboards at finite scales

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### ABSTRACT

Modern engineering designs demand composite materials tailored for application-specific performance. At length scales comparable to the individual microstructural inhomogeneity, the composite behavior is realization dependent and statistical in nature. Understanding and quantifying the transition from a statistical to a deterministic continuum description is important in the context of multiscale mechanics. A promising approach is to use the Hill-Mandel homogeneity condition to determine upper and lower (scale-dependent) bounds. This study aims to calculate the Hill-Mandel bounds and establish scaling laws using checkerboard microstructures, where each cell is either of two isotropic phases. For given volume fractions of phases, a specific realization is obtained and anisotropic stiffness and compliance tensors are determined. Isotropy is recovered by ensemble averaging, with the effective elastic moduli approaching the observed values for a homogeneous continuum. Based on this scaling behavior, the concept of a scaling function is introduced by contracting the stiffness and the compliance tensors. It is postulated that the scaling function depends only on the contrast in the elastic properties and a nondimensional length scale (mesoscale). This will be verified by conducting numerical experiments on 10 material combinations with varying contrasts in elastic properties. It will be demonstrated that the scaling function unifies the length scales and, subsequently, also unifies the treatment of a variety of materials. As a consequence of extensive Monte-Carlo simulations, the statistics of the stiffness and compliance tensors at finite length scales may also be studied through appropriate probability densities and statistical moments.